TRİGONOMETRİ İSPATLAR

1.

- Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\sin a+\sin b+\sin c=4.\cos\frac{a}{2}.\cos\frac{b}{2}.\cos\frac{c}{2}$ olduğunu gösteriniz?

2.

- Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\cos a+\cos b+\cos c=1+4.\sin\frac{a}{2}.\sin\frac{b}{2}.\sin\frac{c}{2}$ olduğunu gösteriniz

3.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\frac{\cos a}{\sin b.\sin c}+\frac{\cos b}{\sin a.\sin c}+\frac{\cos c}{\sin a.\sin b}=2$ olduğunu gösteriniz?

4.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\sin 2a+\sin 2b+\sin 2c=4.\sin a.\sin b.\sin c$ olduğunu gösteriniz?

5.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\frac{\sin 2a+\sin 2b+\sin 2c}{\sin a+\sin b+\sin c}=8\sin\frac{a}{2}\sin\frac{b}{2}\sin\frac{c}{2}$

6.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\sin(b+c-a)+\sin(c+a-b)+\sin(a+b-c)=4\sin a\sin b\sin c$

7.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\cos 2a+\cos 2b+\cos 2c=-1-4.\cos a\cos b\cos c$ olduğunu gösteriniz?

8.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\cos 2a+\cos 2b-\cos 2c=1-4\sin a\sin b\cos c$ olduğunu gösteriniz?

9.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\cos a+\cos b-\cos c=-1+4.\cos\frac{a}{2}.\cos\frac{b}{2}.\sin\frac{c}{2}$

10.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\sin 2a+\sin 2b-\sin 2c=4\cos a\cos b\sin c$

11.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\sin a+\sin b-\sin c=4\sin\frac{a}{2}\sin\frac{b}{2}\cos\frac{c}{2}$

12.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\sin^2 a+\sin^2 b+\sin^2 c=2+2\cos a\cos b\cos c$

13.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\sin^2 a+\sin^2 b-\sin^2 c=2\sin a\sin b\cos c$ olduğunu gösteriniz

14.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\cos^2 a+\cos^2 b+\cos^2 c=1-2\cos a\cos b\cos c$

15.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\cos^2 a+\cos^2 b-\cos^2 c=1-2\sin a\sin b\cos c$

16.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\sin^2\frac{a}{2}+\sin^2\frac{b}{2}+\sin^2\frac{c}{2}=1-2\sin\frac{a}{2}\sin\frac{b}{2}\sin\frac{c}{2}$

17.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\sin^2\frac{a}{2}+\sin^2\frac{b}{2}-\sin^2\frac{c}{2}=\frac{2+2\sin^2\frac{a}{2}-2+2\sin^2\frac{b}{2}-1+1-2\sin^2\frac{c}{2}}{2}$

18.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\tan\frac{a}{2}\tan\frac{b}{2}+\tan\frac{b}{2}\tan\frac{c}{2}+\tan\frac{a}{2}\tan\frac{c}{2}=1$

19.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\tan a+\tan b+\tan c=\tan a\tan b\tan c$ olduğunu gösteriniz

20.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\cot\frac{a}{2}+\cot\frac{b}{2}+\cot\frac{c}{2}=\cot\frac{a}{2}\cot\frac{b}{2}\cot\frac{c}{2}$

21.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\cot b\cot c+\cot c\cot a+\cot a\cot b=1$

22.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\sin(b+2c)+\sin(c+2a)+\sin(a+2b)=-4\sin\frac{b-c}{2}\sin\frac{c-a}{2}\sin\frac{a-b}{2}$

23.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\sin\frac{a}{2}+\sin\frac{b}{2}+\sin\frac{c}{2}-1=4\sin\frac{\pi-a}{4}\sin\frac{\pi-b}{4}\sin\frac{\pi-c}{4}$

24.

- Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\frac{b-c}{b+c}=\frac{\tan\frac{b-c}{2}}{\tan\frac{b+c}{2}}$ olduğunu gösteriniz?

25.

$a+b+c=2s$
$\sin(s-a)\sin(s-b)+\sin s\sin(s-c)=\sin a\sin b$

26.

$a+b+c=2s$
$\sin s\sin(s-a)\sin(s-b)\sin(s-c)=\frac{1}{4}(1-\cos^2 a-\cos^2 b-\cos^2 c+2\cos a\cos b\cos c)$

27.

$a+b+c=2s$
$\sin(s-a)+\sin(s-b)+\sin(s-c)-\sin c=4\sin\frac{a}{2}\sin\frac{b}{2}\sin\frac{c}{2}$

28.

$a+b+c=2s$
$\cos^2 s+\cos^2(s-a)+\cos^2(s-b)+\cos^2(s-c)=2+2\cos a\cos b\cos c$

29.

$a+b+c=2s$
$\cos^2 a+\cos^2 b+\cos^2 c+2\cos a\cos b\cos c=1+4\cos s\cos(s-a)\cos(s-b)\cos(s-c)$

30.

$\alpha+\beta+\omega+\theta=2\pi$
$\cos\alpha+\cos\beta+\cos\omega+\cos\theta+4\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha+\omega}{2}\cos\frac{\alpha+\theta}{2}=0$

31.

$\alpha+\beta+\omega+\theta=2\pi$
$\sin\alpha-\sin\beta+\sin\omega-\sin\theta+4\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha+\omega}{2}\cos\frac{\alpha+\theta}{2}=0$

32.

$1-\cos^2 a-\cos^2 b-\cos^2 c+2\cos a\cos b\cos c=4\sin\frac{a+b+c}{2}\sin\frac{a+b-c}{2}\sin\frac{a-b+c}{2}\sin\frac{-a+b+c}{2}$

ÇÖZÜMLER

1.

- Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*

$\sin a+\sin b+\sin c=4.\cos\frac{a}{2}.\cos\frac{b}{2}.\cos\frac{c}{2}$ olduğunu gösteriniz?

$$\sin a+\sin b+\sin c=\sin a+2\sin\frac{b+c}{2}.\cos\frac{b-c}{2}$$
$$=2\sin\frac{a}{2}.\cos\frac{a}{2}+2\cos\frac{a}{2}.\cos\frac{b-c}{2}$$
$$=2\cos\frac{a}{2}.(\sin\frac{a}{2}+\cos\frac{b-c}{2})$$
$$=2\cos\frac{a}{2}.(\cos\frac{b+c}{2}+\cos\frac{b-c}{2})$$
$$=2\cos\frac{a}{2}.2\cos\frac{b}{2}.\cos\frac{c}{2}$$
$$=4.\cos\frac{a}{2}.\cos\frac{b}{2}.\cos\frac{c}{2}$$

2.

- Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*

$\cos a+\cos b+\cos c=1+4.\sin\frac{a}{2}.\sin\frac{b}{2}.\sin\frac{c}{2}$ olduğunu gösteriniz

$$\cos a+\cos b+\cos c=\cos a+2.\cos\frac{b+c}{2}.\cos\frac{b-c}{2}$$
$$=1-2\sin^2\frac{a}{2}+2.\cos\frac{b+c}{2}.\cos\frac{b-c}{2}$$
$$=1-2\sin^2\frac{a}{2}+2.\cos\frac{b+c}{2}.\cos\frac{b-c}{2}$$
$$=1+2\sin\frac{a}{2}.(\cos\frac{b-c}{2}-\cos\frac{b+c}{2})$$

$\cos\frac{b+c}{2}=\sin\frac{a}{2}$

$$=1+2\sin\frac{a}{2}.-2.\sin\frac{b}{2}.\sin(-\frac{c}{2})$$
$$=1+4.\sin\frac{a}{2}.\sin\frac{b}{2}.\sin\frac{c}{2}$$

3.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*

$\frac{\cos a}{\sin b.\sin c}+\frac{\cos b}{\sin a.\sin c}+\frac{\cos c}{\sin a.\sin b}=2$ olduğunu gösteriniz?

$$\frac{\cos a}{\sin b.\sin c}+\frac{\cos b}{\sin a.\sin c}+\frac{\cos c}{\sin a.\sin b}=\frac{\sin a\cos a+\sin b.\cos b+\sin c.\cos}{\sin a.\sin b.\sin c}$$
$$=\frac{\sin a.\cos a+\frac{1}{2}\sin 2b+\frac{1}{2}\sin 2c}{\sin a.\sin b.\sin c}$$
$$=\frac{\sin a.\cos a+\frac{1}{2}.(\sin 2b+\sin 2c)}{\sin a\sin b.\sin c}$$
$$=\frac{\sin a.\cos a+\frac{1}{2}.2.\sin(b+c).\cos(b-c)}{\sin a.\sin b.\sin c}$$

$\cos a=-\cos(b+c)$

$$=\frac{\sin a.(\cos a+\cos(b-c))}{\sin a\sin b.\sin c}$$
$$=\frac{\cos(b-c)-\cos(b+c)}{\sin b.\sin c}$$
$$=\frac{-2\sin b.\sin(-c)}{\sin b.\sin c}$$
$$=2$$

4.

- Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*

$\sin 2a+\sin 2b+\sin 2c=4.\sin a.\sin b.\sin c$ olduğunu gösteriniz?

$$\sin 2a+\sin 2b+\sin 2c=2\sin a.\cos a+2\sin(b+c).\cos(b-c)$$
$$=2\sin a(\cos a+\cos(b-c))$$
$$=2\sin a(\cos(b-c)-\cos(b+c))$$
$$=2\sin a-2.\sin b.\sin(-c)$$
$$=4.\sin a.\sin b.\sin c$$

$a+b+c=180$ | $a+b+c=180$
$b+c=180-a$ | $b+c=180-a$
$\sin(b+c)=\sin a$ | $\cos(b+c)=-\cos a$

5.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*

$\frac{\sin 2a+\sin 2b+\sin 2c}{\sin a+\sin b+\sin c}=8\sin\frac{a}{2}\sin\frac{b}{2}\sin\frac{c}{2}$

$\sin 2a+\sin 2b+\sin 2c=4.\sin a.\sin b.\sin c$

$\sin a+\sin b+\sin c=4.\cos\frac{a}{2}.\cos\frac{b}{2}.\cos\frac{c}{2}$

$$\frac{\sin 2a+\sin 2b+\sin 2c}{\sin a+\sin b+\sin c}=\frac{4.\sin a\sin b\sin c}{4\cos\frac{a}{2}\cos\frac{b}{2}\cos\frac{c}{2}}$$
$$=\frac{2.2.2\sin\frac{a}{2}\cos\frac{a}{2}\sin\frac{b}{2}\cos\frac{b}{2}\sin\frac{c}{2}\cos\frac{c}{2}}{\cos\frac{a}{2}\cos\frac{b}{2}\cos\frac{c}{2}}$$
$$=8\sin\frac{a}{2}\sin\frac{b}{2}\sin\frac{c}{2}$$

6.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*

$\sin(b+c-a)+\sin(c+a-b)+\sin(a+b-c)=4\sin a\sin b\sin c$

$a+b+c=180$

$b+c=180-a$

$b+c-a=180-2a$

$\sin(b+c-a)=\sin 2a \quad \sin(c+a-b)=\sin 2b \quad \sin(a+b-c)=\sin 2c$

$\sin 2a+\sin 2b+\sin 2c=4.\sin a.\sin b.\sin c$

7.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*

$\cos 2a+\cos 2b+\cos 2c=-1-4.\cos a\cos b\cos c$ olduğunu gösteriniz?

$$\begin{aligned}\cos 2a+\cos 2b+\cos 2c&=2\cos^2 a-1+2(\cos(b+c)\cos(b-c))\\&=-1+2\cos a(\cos a-\cos(b-c))\\&=-1-2\cos a(\cos(b+c)+\cos(b-c))\\&=-1-2\cos a.2(\cos b.\cos c)\\&=-1-4.\cos a\cos b\cos c\end{aligned}$$

$a+b+c=180$

$b+c=180-a$

$\cos(b+c)=-\cos a$

8.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*

$\cos 2a+\cos 2b-\cos 2c=1-4\sin a\sin b\cos c$ olduğunu gösteriniz?

$$\begin{aligned}\cos 2a+\cos 2b-\cos 2c&=1-2\sin^2 a-2(\sin(b+c)\sin(b-c))\\&=1-2\sin a(\sin a+\sin(b-c))\\&=1-2\sin a.2(\sin(b+c)+\sin(b-c))\\&=1-4\sin a\sin b\cos c\end{aligned}$$

$a+b+c=180$

$b+c=180-a$

$\sin(b+c)=\sin a$

9.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*

$\cos a+\cos b-\cos c=-1+4.\cos\frac{a}{2}.\cos\frac{b}{2}.\sin\frac{c}{2}$

$$\begin{aligned}\cos a+\cos b-\cos c&=2\cos^2\frac{a}{2}-1+2\sin\frac{b+c}{2}\sin\frac{c-b}{2}\\&=2\cos^2\frac{a}{2}-1+2\cos\frac{a}{2}\sin\frac{c-b}{2}\\&=2\cos\frac{a}{2}\left(\cos\frac{a}{2}+\sin\frac{c-b}{2}\right)-1\\&=2\cos\frac{a}{2}\left(\sin\frac{b+c}{2}+\sin\frac{c-b}{2}\right)-1\\&=2\cos\frac{a}{2}2\sin\frac{c}{2}\cos\frac{b}{2}-1\\&=-1+4\cos\frac{a}{2}\cos\frac{b}{2}\sin\frac{c}{2}\end{aligned}$$

$\sin\frac{b+c}{2}=\cos\frac{a}{2}$

10.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*

$\sin 2a+\sin 2b-\sin 2c=4\cos a\cos b\sin c$

$$\begin{aligned}\sin 2a+\sin 2b-\sin 2c&=2\sin a\cos a+2(\cos(b+c)\sin(b-c))\\&=2\cos a(\sin a-\sin(b-c))\\&=2\cos a(\sin(b+c)-\sin(b-c))\\&=2\cos a.2\cos b\sin c\\&=4\cos a\cos b\sin c\end{aligned}$$

$a+b+c=180$	$a+b+c=180$
$b+c=180-a$	$b+c=180-a$
$\cos(b+c)=-\cos a$	$\sin(b+c)=\sin a$

11.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*

$\sin a+\sin b-\sin c=4\sin\frac{a}{2}\sin\frac{b}{2}\cos\frac{c}{2}$

$$\begin{aligned}\sin a+\sin b-\sin c&=2\sin\frac{a}{2}\cos\frac{a}{2}+2\left(\cos\frac{b+c}{2}\sin\frac{b-c}{2}\right)\\&=2\sin\frac{a}{2}\cos\frac{a}{2}+2\left(\sin\frac{a}{2}\sin\frac{b-c}{2}\right)\\&=2\sin\frac{a}{2}\left(\cos\frac{a}{2}+\sin\frac{b-c}{2}\right)\\&=2\sin\frac{a}{2}\left(\sin\frac{b+c}{2}+\sin\frac{b-c}{2}\right)\\&=4\sin\frac{a}{2}\sin\frac{b}{2}\cos\frac{c}{2}\end{aligned}$$

12.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*

$\sin^2 a+\sin^2 b+\sin^2 c=2+2\cos a\cos b\cos c$

$$\begin{aligned}\sin^2 a+\sin^2 b+\sin^2 c&=\frac{2\sin^2 a+2\sin^2 b+2\sin^2 c}{2}\\&=\frac{2\sin^2 a+1-\cos 2b+1-\cos 2c}{2}\\&=\frac{2\sin^2 a+2-(\cos 2b+\cos 2c)}{2}\\&=\frac{2\sin^2 a+2-2\cos(b+c)\cos(b-c)}{2}\\&=\frac{2-2\cos^2 a+2-2\cos(b+c)\cos(b-c)}{2}\\&=2+\cos a(\cos(b-c)+\cos(b+c))\\&=2+\cos a.2.\cos b.\cos c\\&=2+2\cos a\cos b\cos c\end{aligned}$$

$1-2\sin^2 b=\cos 2b$

$\cos a=\cos(180-(b+c))=-\cos(b+c)$

13.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\sin^2 a+\sin^2 b-\sin^2 c=2\sin a\sin b\cos c$ olduğunu gösteriniz

$$\sin^2 a+\sin^2 b-\sin^2 c=\frac{2\sin^2 a+2\sin^2 b-2\sin^2 c}{2}$$
$$=\frac{2\sin^2 a+2\sin^2 b-1+1-2\sin^2 c}{2}$$
$$=\frac{2\sin^2 a-\cos 2b+\cos 2c}{2}$$
$$=\frac{2\sin^2 a-2(\sin(b+c)\sin(c-b)}{2}$$
$$=\frac{2\sin^2 a+2(\sin a\sin(b-c))}{2}$$
$$=\sin a(\sin a+\sin(b-c))$$
$$=\sin a(\sin(b+c)+\sin(b-c))$$
$$=2\sin a\sin b\cos c$$

14.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\cos^2 a+\cos^2 b+\cos^2 c=1-2\cos a\cos b\cos c$

$$\cos^2 a+\cos^2 b+\cos^2 c=\frac{2\cos^2 a+2\cos^2 b+2\cos^2 c}{2}$$

$a+b+c=180$
$b+c=180-a$
$\cos(b+c)=-\cos a$

$$=\frac{2+2\cos^2 a+2\cos^2 b-1+2\cos^2 c-1}{2}$$
$$=\frac{2+2\cos^2 a+\cos 2b+\cos 2c}{2}$$
$$=\frac{2+2\cos^2 a+2(\cos(b+c)\cos(b-c))}{2}$$
$$=1+\cos^2 a+(\cos(b+c)\cos(b-c))$$
$$=1+\cos a(-\cos a\cos(b-c))$$
$$=1-\cos a(\cos(b+c)+\cos(b-c))$$
$$=1-2\cos a\cos b\cos c$$

15.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\cos^2 a+\cos^2 b-\cos^2 c=1-2\sin a\sin b\cos c$

$$\cos^2 a+\cos^2 b-\cos^2 c=\frac{2\cos^2 a+2\cos^2 b-2\cos^2 c}{2}$$
$$=\frac{2\cos^2 a+2\cos^2 b-1+1-2\cos^2 c}{2}$$
$$=\frac{2\cos^2 a+\cos 2b-\cos 2c}{2}$$
$$=\frac{2\cos^2 a-2(\sin(b+c)\sin(b-c))}{2}$$
$$=1-\sin^2 a-(\sin a\sin(b-c))$$
$$=1-\sin a(\sin a+\sin(b-c))$$
$$=1-\sin a(\sin(b+c)+\sin(b-c))$$
$$=1-2\sin a\sin b\cos c$$

16.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*
$\sin^2\frac{a}{2}+\sin^2\frac{b}{2}+\sin^2\frac{c}{2}=1-2\sin\frac{a}{2}\sin\frac{b}{2}\sin\frac{c}{2}$

$$\sin^2\frac{a}{2}+\sin^2\frac{b}{2}+\sin^2\frac{c}{2}=\frac{2\sin^2\frac{a}{2}+2\sin^2\frac{b}{2}+2\sin^2\frac{c}{2}}{2}$$
$$=\frac{2+2\sin^2\frac{a}{2}-1+2\sin^2\frac{b}{2}-1+2\sin^2\frac{c}{2}}{2}$$
$$=\frac{2+2\sin^2\frac{a}{2}-(\cos b+\cos c)}{2}$$
$$=\frac{2+2\sin^2\frac{a}{2}-2\left(\cos\frac{b+c}{2}+\cos\frac{b-c}{2}\right)}{2}$$
$$=1+\sin^2\frac{a}{2}-\left(\sin\frac{a}{2}+\cos\frac{b-c}{2}\right)$$
$$=1+\sin\frac{a}{2}\left(\sin\frac{a}{2}-\cos\frac{b-c}{2}\right)$$
$$=1+\sin\frac{a}{2}\left(\cos\frac{b+c}{2}-\cos\frac{b-c}{2}\right)$$
$$=1-2\sin\frac{a}{2}\sin\frac{b}{2}\sin\frac{c}{2}$$

17.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*

$$\sin^2\frac{a}{2}+\sin^2\frac{b}{2}-\sin^2\frac{c}{2}=\frac{2+2\sin^2\frac{a}{2}-2+2\sin^2\frac{b}{2}-1+1-2\sin^2\frac{c}{2}}{2}$$
$$=\frac{2-2\cos^2\frac{a}{2}-\cos b+\cos c}{2}$$
$$=\frac{2-2\cos^2\frac{a}{2}+2\left(\sin\frac{b+c}{2}\sin\frac{b-c}{2}\right)}{2}$$
$$=1-\cos^2\frac{a}{2}+\left(\cos\frac{a}{2}\sin\frac{b-c}{2}\right)$$
$$=1-\cos\frac{a}{2}\left(\cos\frac{a}{2}-\sin\frac{b-c}{2}\right)$$
$$=1-\cos\frac{a}{2}\left(\sin\frac{b+c}{2}-\sin\frac{b-c}{2}\right)$$
$$=1-2\cos\frac{a}{2}\cos\frac{b}{2}\sin\frac{c}{2}$$

18.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*

$\tan\frac{a}{2}\tan\frac{b}{2}+\tan\frac{b}{2}\tan\frac{c}{2}+\tan\frac{a}{2}\tan\frac{c}{2}=1$

$\frac{a+b+c}{2}=90$

$\frac{a+b}{2}=90-\frac{c}{2}$

$\tan\left(\frac{a+b}{2}\right)=\cot\frac{c}{2}$

$\frac{\tan\frac{a}{2}+\tan\frac{b}{2}}{1-\tan\frac{a}{2}\tan\frac{b}{2}}=\frac{1}{\tan\frac{c}{2}}$

$\tan\frac{c}{2}\left(\tan\frac{a}{2}+\tan\frac{b}{2}\right)=1-\tan\frac{a}{2}\tan\frac{b}{2}$

$\tan\frac{a}{2}\tan\frac{b}{2}+\tan\frac{a}{2}\tan\frac{c}{2}+\tan\frac{b}{2}\tan\frac{c}{2}=1$

19.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*

$\tan a+\tan b+\tan c=\tan a\tan b\tan c$ olduğunu gösteriniz

$a+b+c=180$

$a+b=180-c$

$\tan(a+b)=-\tan c$

$\frac{\tan a+\tan b}{1-\tan a\tan b}=-\tan c$

$\tan a+\tan b=-\tan c(1-\tan a\tan b)$

$\tan a+\tan b=-\tan c+\tan a\tan b\tan c$

$\tan a+\tan b+\tan c=\tan a\tan b\tan c$

20.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*

$\cot\frac{a}{2}+\cot\frac{b}{2}+\cot\frac{c}{2}=\cot\frac{a}{2}\cot\frac{b}{2}\cot\frac{c}{2}$

$\frac{a+b+c}{2}=90$

$\frac{a+b}{2}=90-\frac{c}{2}$

$\cot\left(\frac{a+b}{2}\right)=\tan\frac{c}{2}$

$\frac{\cot\frac{a}{2}\cot\frac{b}{2}-1}{\cot\frac{a}{2}+\cot\frac{b}{2}}=\frac{1}{\cot\frac{c}{2}}$

$\cot\frac{c}{2}\left(\cot\frac{a}{2}\cot\frac{b}{2}-1\right)=\cot\frac{a}{2}+\cot\frac{b}{2}$

$\cot\frac{a}{2}+\cot\frac{b}{2}+\cot\frac{c}{2}=\cot\frac{a}{2}\cot\frac{b}{2}\cot\frac{c}{2}$

21.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*

$\cot b\cot c+\cot c\cot a+\cot a\cot b=1$

$a+b+c=180$

$a+b=180-c$

$\cot(a+b)=-\cot c$

$\frac{\cot a\cot b-1}{\cot a+\cot b}=-\cot c$

$\cot a\cot b-1=-\cot c(\cot a+\cot b)$

$\cot a\cot b+\cot a\cot c+\cot b\cot c=1$

22.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*

$\sin(b+2c)+\sin(c+2a)+\sin(a+2b)=-4\sin\frac{b-c}{2}\sin\frac{c-a}{2}\sin\frac{a-b}{2}$

$\sin(b+2c)+\sin(c+2a)+\sin(a+2b)=\sin(c-a)+\sin(a-b)+\sin(b-c)$

$=2\sin\frac{c-a}{2}\cos\frac{c-a}{2}+2\left(\sin\frac{a-c}{2}\cos\frac{a-2b+c}{2}\right)$

$=2\sin\frac{c-a}{2}\left(\cos\frac{c-a}{2}-\cos\frac{a-2b+c}{2}\right)$

$=2\sin\frac{c-a}{2}\left(\cos\frac{a-c}{2}-\cos\frac{a-2b+c}{2}\right)$

$=2\sin\frac{c-a}{2}\left(-2\sin\frac{a-b}{2}\sin\frac{b-c}{2}\right)$

$=-4\sin\frac{b-c}{2}\sin\frac{c-a}{2}\sin\frac{a-b}{2}$

23.

Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*

$\sin\frac{a}{2}+\sin\frac{b}{2}+\sin\frac{c}{2}-1=4\sin\frac{\pi-a}{4}\sin\frac{\pi-b}{4}\sin\frac{\pi-c}{4}$

$\sin\frac{a}{2}+\sin\frac{b}{2}+\sin\frac{c}{2}-1=\cos\frac{b+c}{2}-1+2\left(\sin\frac{b+c}{4}\cos\frac{b-c}{4}\right)$

$=1-2\sin^2\frac{b+c}{4}-1+2\left(\sin\frac{b+c}{4}\cos\frac{b-c}{4}\right)$

$=-2\sin^2\frac{b+c}{4}+2\left(\sin\frac{b+c}{4}\cos\frac{b-c}{4}\right)$

$=2\sin\frac{b+c}{4}\left(-\sin\frac{b+c}{4}+\cos\frac{b-c}{4}\right)$

$=2\sin\frac{b+c}{4}\left(\cos\left(\frac{\pi}{2}+\frac{b+c}{4}\right)+\cos\frac{b-c}{4}\right)$

$=2\sin\frac{b+c}{4}2.\cos\left(\frac{\pi}{4}+\frac{b}{4}\right)\cos\left(\frac{\pi}{4}+\frac{c}{4}\right)$

$=4\sin\frac{\pi-a}{4}\sin\left(\frac{\pi}{4}-\frac{b}{4}\right)\sin\left(\frac{\pi}{4}-\frac{c}{4}\right)$

$=4\sin\frac{\pi-a}{4}\sin\frac{\pi-b}{4}\sin\frac{\pi-c}{4}$

24.

- Bir ABC üçgeninde $a+b+c=180$ *olmak üzere*

$$\frac{b-c}{b+c}=\frac{\tan\frac{b-c}{2}}{\tan\frac{b+c}{2}}$$ olduğunu gösteriniz?

Sinüs teoreminden

$$\frac{b}{\sin b}=\frac{c}{\sin c}=2R \quad ise \quad b=2R\sin b \quad ve \quad c=2R\sin c$$

$$\frac{b-c}{b+c}=\frac{2R(\sin b-\sin c)}{2R(\sin b+\sin c)}$$

$$=\frac{2\cos\frac{b+c}{2}.\sin\frac{b-c}{2}}{2\sin\frac{b+c}{2}.\cos\frac{b-c}{2}}$$

$$=\frac{\tan\frac{b-c}{2}}{\tan\frac{b+c}{2}}$$

$$\frac{b-c}{b+c}=\frac{\tan\frac{b-c}{2}}{\tan\frac{b+c}{2}}$$

25.

$$a+b+c=2s$$

$$\sin(s-a)\sin(s-b)+\sin s\sin(s-c)=\sin a\sin b$$

$$\sin(s-a)\sin(s-b)+\sin s\sin(s-c\quad)$$

$$=-\frac{1}{2}(\cos(2s-a-b)-\cos(b-a)+\cos(2s-c)-\cos c)$$

$$=-\frac{1}{2}(\cos(a+b+c-a-b)-\cos(c)+\cos(a+b+c-c)-\cos(b-a))$$

$$=-\frac{1}{2}.-2\sin b.\sin a$$

$$=\sin a\sin b$$

26.

$$a+b+c=2s$$

$$\sin s\sin(s-a)\sin(s-b)\sin(s-c)=\frac{1}{4}(1-\cos^2a-\cos^2b-\cos^2c+2\cos a\cos b\cos c)$$

$$\sin s\sin(s-a)\sin(s-b)\sin(s-c)=\left(-\frac{1}{2}(\cos(2s-a)-\cos a)\right)\left(-\frac{1}{2}(\cos(2s-b-c)-\cos(c-b))\right)$$

$$=\frac{1}{4}(\cos(b+c)-\cos a)(\cos a-\cos(c-b))$$

$$=\frac{1}{4}(\cos(b+c)\cos a-\cos(c-b)\cos(b+c)-\cos^2a+\cos a\cos(c-b))$$

$$=\frac{1}{8}((\cos(a+b+c)+\cos(b+c-a))-(\cos 2c+\cos 2b)-2\cos^2a+(\cos(a+c-b)+\cos(a-c+b)))$$

$$=\frac{1}{8}(2-2\cos^2a-2\cos^2b-2\cos^2c+\cos(a+b+c)+\cos(a+c-b)+\cos(b+c-a)+\cos(a-c+b))$$

$$=\frac{1}{8}(2-2\cos^2a-2\cos^2b-2\cos^2c+\cos(a+b+c)+\cos(a+c-b)+\cos(b+c-a)+\cos(a-c+b))$$

$$=\frac{1}{8}(2-2\cos^2a-2\cos^2b-2\cos^2c+2(\cos(a+c)\cos b+\cos b+\cos(c-a)))$$

$$=\frac{1}{8}(2-2\cos^2a-2\cos^2b-2\cos^2c+2\cos b(\cos(a+c)+\cos(c-a)))$$

$$=\frac{1}{8}(2-2\cos^2a-2\cos^2b-2\cos^2c+4\cos b\cos a\cos c)$$

$$=\frac{1}{4}(1-\cos^2a-\cos^2b-\cos^2c+2\cos a\cos b\cos c\quad)$$

27.

$$a+b+c=2s$$

$$\sin(s-a)+\sin(s-b)+\sin(s-c)-\sin c=4\sin\frac{a}{2}\sin\frac{b}{2}\sin\frac{c}{2}$$

$$\sin(s-a)+\sin(s-b)+\sin(s-c)-\sin s$$

$$=2\sin\frac{2s-a-b}{2}\cos\frac{b-a}{2}+2\cos\frac{2s-c}{2}\sin\left(-\frac{c}{2}\right)$$

$$=2\sin\frac{c}{2}\cos\frac{b-a}{2}+2\cos\frac{a+b}{2}\sin\frac{c}{2}$$

$$=2\sin\frac{c}{2}\left(\cos\frac{b-a}{2}-\cos\frac{a+b}{2}\right)$$

$$=4\sin\frac{a}{2}\sin\frac{b}{2}\sin\frac{c}{2}$$

28.

$$a+b+c=2s$$

$$\cos^2s+\cos^2(s-a)+\cos^2(s-b)+\cos^2(s-c)=2+2\cos a\cos b\cos c$$

$$\cos^2s+\cos^2(s-a)+\cos^2(s-b)+\cos^2(s-c):$$

$$=\frac{1+\cos\frac{s}{2}}{2}+\frac{1+\cos\frac{s-a}{2}}{2}+\frac{1+\cos\frac{s-b}{2}}{2}+\frac{1+\cos\frac{s-c}{2}}{2}$$

$$=2+\frac{1}{2}\left(\cos\frac{s}{2}+\cos\frac{s-a}{2}+\cos\frac{s-b}{2}+\cos\frac{s-c}{2}\right)$$

$$=2+\cos\frac{2s-a}{2}\cos\frac{a}{2}+\cos\frac{2s-b-c}{2}\cos\frac{c-b}{2}$$

$$=2+\cos\frac{a}{2}\left(\cos\frac{b+c}{2}+\cos\frac{c-b}{2}\right)$$

$$=2+2\cos a\cos b\cos c$$

29.

$$a+b+c=2s$$
$$\cos^2 a+\cos^2 b+\cos^2 c+2\cos a\cos b\cos c=1+4\cos s\cos(s-a)\cos(s-b)\cos(s-c)$$

$$\cos^2 a+\cos^2 b+\cos^2 c+2\cos a\cos b\cos c$$
$$=\frac{2\cos^2 a+2\cos^2 b+2\cos^2 c}{2}+2\cos a\cos b\cos c$$
$$=\frac{2+2\cos^2 a+2\cos^2 b-1+2\cos^2 c-1}{2}+2\cos a\cos b\cos c$$
$$=\frac{2+2\cos^2 a+\cos 2b+\cos 2c}{2}+2\cos a\cos b\cos c$$
$$=\frac{2+2\cos^2 a+2\left(\cos(b+c)\cos(b-c)\right)}{2}+2\cos a\cos b\cos c$$
$$=1+\cos^2 a+\left(\cos(b+c)\cos(b-c)\right)+2\cos a\cos b\cos c$$
$$=1+\cos a\left(\cos a+2\cos b\cos c\right)+\left(\cos(b+c)\cos(b-c)\right)$$
$$=1+\cos a\left(\cos a+2\frac{1}{2}\cos(b+c)+\cos(b-c)\right)+\left(\cos(b+c)\cos(b-c)\right)$$
$$=1+\cos a\left(\cos a+\cos(b+c)+\cos(b-c)\right)+\left(\cos(b+c)\cos(b-c)\right)$$
$$=1+\cos^2 a+\cos a\cos(b+c)+\cos a\cos(b-c)+\left(\cos(b+c)\cos(b-c)\right)$$
$$=1+\cos(b+c)\left(\cos a+\cos(b-c)\right)+\cos a\left(\cos a+\cos(b+c)\right)$$
$$=1+\left(\cos a+\cos(b-c)\right)\left(\cos a+\cos(b+c)\right)$$
$$=1+2\cos\frac{a+b-c}{2}.\cos\frac{a+c-b}{2}2.\cos\frac{a+b+c}{2}\cos\frac{a-b-c}{2}$$
$$=1+4\cos\frac{a+b+c}{2}\cos\frac{b+c-a}{2}\cos\frac{a+c-b}{2}\cos\frac{a+b-c}{2}$$
$$=1+4\cos s\cos(s-a)\cos(s-b)\cos(s-c)$$

30.

$$\alpha+\beta+\omega+\theta=2\pi$$
$$\cos\alpha+\cos\beta+\cos\omega+\cos\theta+4\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha+\omega}{2}\cos\frac{\alpha+\theta}{2}=0$$

$$\alpha+\beta+\omega+\theta=2\pi$$
$$\alpha+\beta=2\pi-\omega-\theta$$
$$\frac{\alpha+\beta}{2}=\pi-\frac{\omega+\theta}{2}$$
$$\cos\frac{\alpha+\beta}{2}=-\cos\frac{\omega+\theta}{2}$$

$$\alpha+\beta+\omega+\theta=2\pi$$
$$\beta+\theta=2\pi-\alpha-\omega$$
$$\frac{\alpha+\omega-\beta-\theta}{4}=\frac{2\alpha+2\omega-2\pi}{4}$$
$$\sin\left(\frac{\alpha+\omega-\beta-\theta}{4}\right)=\sin\left(\frac{\alpha+\omega}{2}-\frac{\pi}{2}\right)=-\cos\frac{\alpha+\omega}{2}$$
$$\sin\left(\frac{\alpha+\theta-\omega-\beta}{4}\right)=\sin\left(\frac{\alpha+\theta}{2}-\frac{\pi}{2}\right)=-\cos\frac{\alpha+\theta}{2}$$

$$2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}+2\cos\frac{\omega+\theta}{2}\cos\frac{\omega-\theta}{2}+4\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha+\omega}{2}\cos\frac{\alpha+\theta}{2}$$
$$=2\cos\frac{\alpha+\beta}{2}\left(\cos\frac{\alpha-\beta}{2}-\cos\frac{\omega-\theta}{2}\right)+4\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha+\omega}{2}\cos\frac{\alpha+\theta}{2}$$
$$=-2.2\cos\frac{\alpha+\beta}{2}\left(\sin\frac{\alpha-\beta+\omega-\theta}{4}\sin\frac{\alpha-\beta-\omega+\theta}{4}\right)+4\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha+\omega}{2}\cos\frac{\alpha+\theta}{2}$$
$$=-4\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha+\omega}{2}\cos\frac{\alpha+\theta}{2}+4\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha+\omega}{2}\cos\frac{\alpha+\theta}{2}$$
$$=0$$

31.

$$\alpha+\beta+\omega+\theta=2\pi$$
$$\sin\alpha-\sin\beta+\sin\omega-\sin\theta+4\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha+\omega}{2}\cos\frac{\alpha+\theta}{2}=0$$

$$\sin\alpha-\sin\beta+\sin\omega-\sin\theta+4\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha+\omega}{2}\cos\frac{\alpha+\theta}{2}$$
$$=2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}+2\cos\frac{\omega+\theta}{2}\sin\frac{\omega-\theta}{2}+4\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha+\omega}{2}\cos\frac{\alpha+\theta}{2}$$
$$=2\cos\frac{\alpha+\beta}{2}\left(\sin\frac{\alpha-\beta}{2}-\sin\frac{\omega-\theta}{2}\right)+4\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha+\omega}{2}\cos\frac{\alpha+\theta}{2}$$
$$=2.2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta+\omega-\theta}{4}\sin\frac{\alpha-\beta-\omega+\theta}{4}+4\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha+\omega}{2}\cos\frac{\alpha+\theta}{2}$$
$$=-4\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha+\omega}{2}\cos\frac{\alpha+\theta}{2}+4\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha+\omega}{2}\cos\frac{\alpha+\theta}{2}$$
$$=0$$

32.

$$1-\cos^2 a-\cos^2 b-\cos^2 c+2\cos a\cos b\cos c=4\sin\frac{a+b+c}{2}\sin\frac{a+b-c}{2}\sin\frac{a-b+c}{2}\sin\frac{-a+b+c}{2}$$

$$1-\cos^2 a-\cos^2 b-\cos^2 c+2\cos a\cos b\cos c$$
$$=1-\frac{2\cos^2 a+2\cos^2 b+2\cos^2 c}{2}+2\cos a\cos b\cos c$$
$$=1-\frac{2+2\cos^2 a+2\cos^2 b-1+2\cos^2 c-1}{2}+2\cos a\cos b\cos c$$
$$=1-\frac{2+2\cos^2 a+\cos 2b+\cos 2c}{2}+2\cos a\cos b\cos c$$
$$=1-\frac{2+2\cos^2 a+2\left(\cos(b+c)\cos(b-c)\right)}{2}+2\cos a\cos b\cos c$$
$$=1-1-\cos^2 a-\left(\cos(b+c)\cos(b-c)\right)+2\cos a\cos b\cos c$$
$$=\cos a\left(-\cos a+2\cos b\cos c\right)-\left(\cos(b+c)\cos(b-c)\right)$$
$$=\cos a\left(-\cos a+2\frac{1}{2}\cos(b+c)+\cos(b-c)\right)-\left(\cos(b+c)\cos(b-c)\right)$$
$$=\cos a\left(-\cos a+\cos(b+c)+\cos(b-c)\right)-\left(\cos(b+c)\cos(b-c)\right)$$
$$=-\cos^2 a+\cos a\cos(b+c)+\cos a\cos(b-c)-\left(\cos(b+c)\cos(b-c)\right)$$
$$=\cos(b+c)\left(\cos a-\cos(b-c)\right)+\cos a\left(-\cos a+\cos(b+c)\right)$$
$$=\left(\cos a-\cos(b-c)\right)\left(\cos(b+c)-\cos a\right)$$
$$=-2\sin\frac{a+b-c}{2}.\sin\frac{a+c-b}{2}-2.\sin\frac{a+b+c}{2}\sin\frac{a-b-c}{2}$$
$$=4\sin\frac{a+b+c}{2}\sin\frac{b+c-a}{2}\sin\frac{a+c-b}{2}\sin\frac{a+b-c}{2}$$